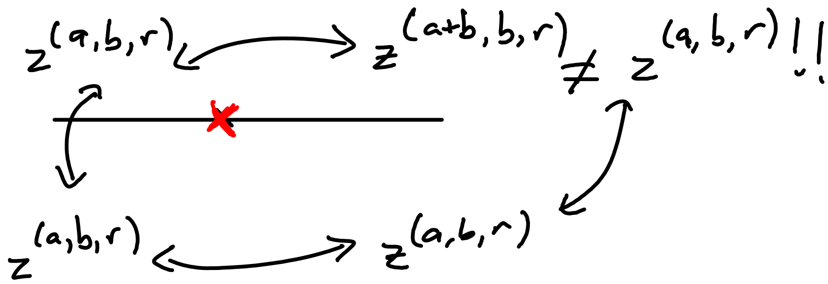
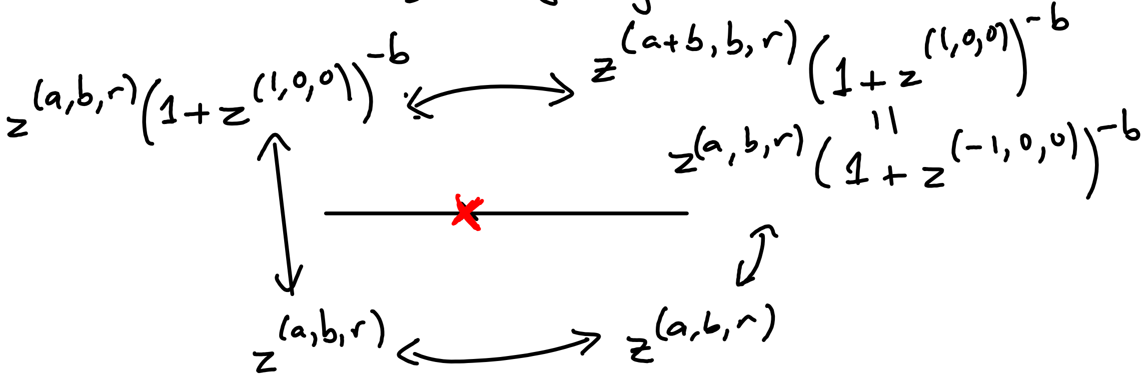
M. Gross – 27/5/08 – Lecture 2

- <u>Singularities</u>: consider a singularity in 2D case

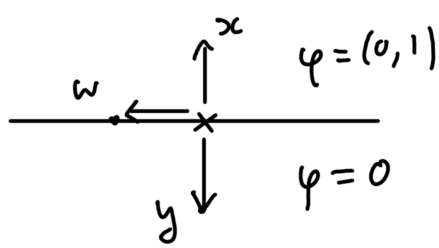
  <u>Problem</u>: gluings are not compatible anymore!



<u>Solution</u>: modify the gluing!



$\rightarrow$ modified gluing of rings: take $\begin{cases} \varphi = (0,1) & \text{on upper half} \\ \varphi = 0 & \text{on lower half} \end{cases}$



Upper ring: $k[x,y,w^{\pm1}]/(y^k)$

Lower ring: $k[x,y,w^{\pm1}]/(x^k)$

<u>Glue as</u>: $k[x,y,w^{\pm1}]/(y^k) \underset{\left(k[x,y,w^{\pm1}]/(x^k,y^k)\right)_{1+w}}{\times} k[x,y,w^{\pm1}]/(x^k)$

with gluing maps $(x,y,w) \mapsto (x,y,w)$ on left

& on right
$$x \mapsto x/(1+w)$$
$$y \mapsto y(1+w)$$
$$w \mapsto w$$

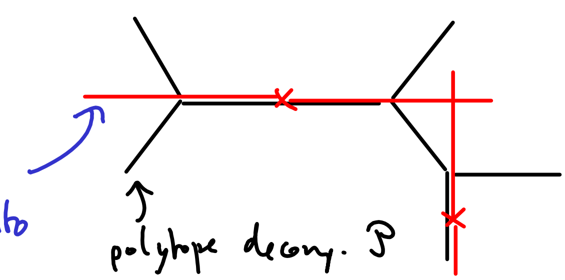
get $\boxed{k[X, Y, W, T]/\langle XY - (1+W)T,\ T^k \rangle}$

where
$$\begin{cases} X = (x, x(1+w)) \\ Y = (y(1+w), y) \\ W = (w, w) \\ T = (xy, xy) \end{cases}$$

Can build explicitly a SLag fibration that behaves this way.

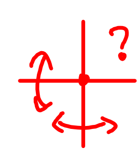
- <u>Idea</u>: attach these gluing automorphisms along lines emanating from singularities, along invariant direction (w in example)

- First issue: extend lines past boundary of cell of decomposition

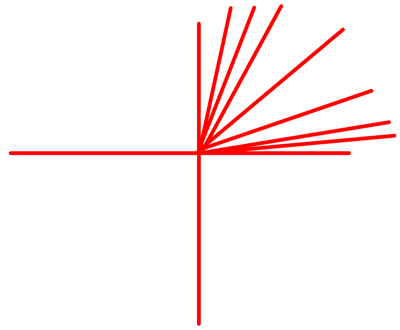


when we go into next cell,

lemma last time: order of vanishing of z^{mr} <u>increases</u> upon transport in direction $-m$ $\Rightarrow$ gluing definition extends past boundary of cell ✓

- More important: when 2 lines hit, noncommutativity of gluing 

$\Rightarrow$ <u>Kontsevich-Soibelman</u>: new rays emanate from the intersection get in fact



union of walls propagating all over (infinite!)
(= union of all tropical trees
= in mirror, union of all exceptional holom. discs
Conj. with ∂ in Lag fiber).

(For now, as a formal space).

Main Thm: Given B $\mathbb{Z}$-affine mfld w/ singularities, $\mathscr{P}$ polyhedral decomposition, φ multivalued strictly convex PL function, and given some local conditions on sings. of B ("local rigidity"), $\exists$ degeneration of CY varieties $\mathfrak{X} \to \operatorname{Spec} k[[t]]$ controlled by this data. The degeneration is uniquely determined by some initial data ("log structure" on sing. fiber $\mathfrak{X}_0$), described by a union of affine hyperplanes with attached automorphisms, containing all tropical trees.

Rmk: • This implies existence of a flat deformation of central fiber in the analytic category (over a disk). The general fiber will be a compactification of $X(\check{B}_0)$, where $(\check{B}, \check{\varphi})$ is the discrete Legendre transform of (B, φ); in particular $X(\check{B}_0) \underset{top.}{\simeq} \check{X}(B_0)$

- It is the Legendre transform that allows us to replace Fukaya / K-S's trees of gradient flow lines with easier tropical curves $\rightsquigarrow$ can push method to all dimensions

- Because $\mathfrak{X} \to \operatorname{Spec} k[[t]]$ is described via tropical trees, tropical rational curves emerge naturally in the calculation of <u>periods</u> of the complex structure

  $\Rightarrow$ establish a direct connection between the B side of mirror symmetry and tropical geometry.

- What we get is a <u>curve</u> in the moduli space of CY's near the LCSL; this curve is a straight line in <u>canonical coordinates</u> on moduli space.